MODEL-INDEPENDENT VELOCITY AND ACCELERATION OF HELIOSPHERIC DISTURBANCES

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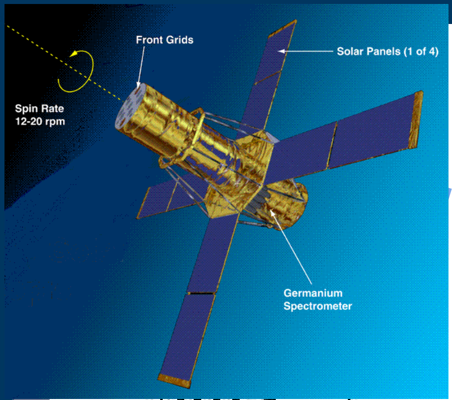
Prologue…

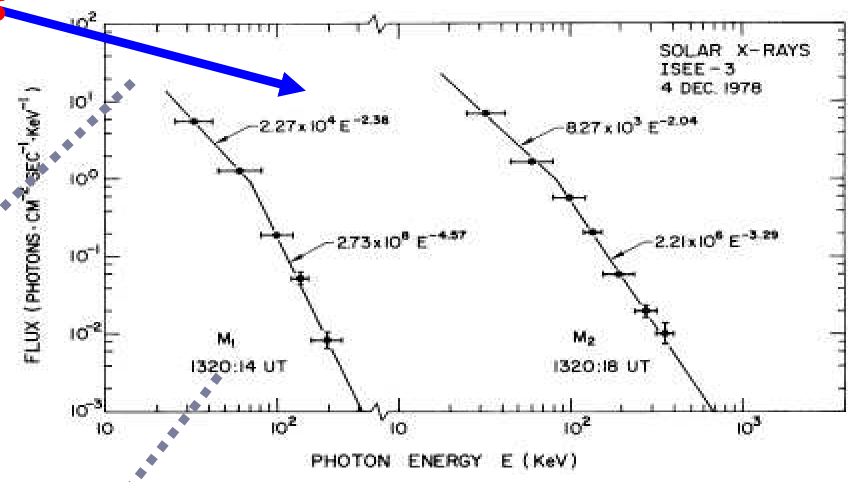
"Acceleration errors are difficult…"

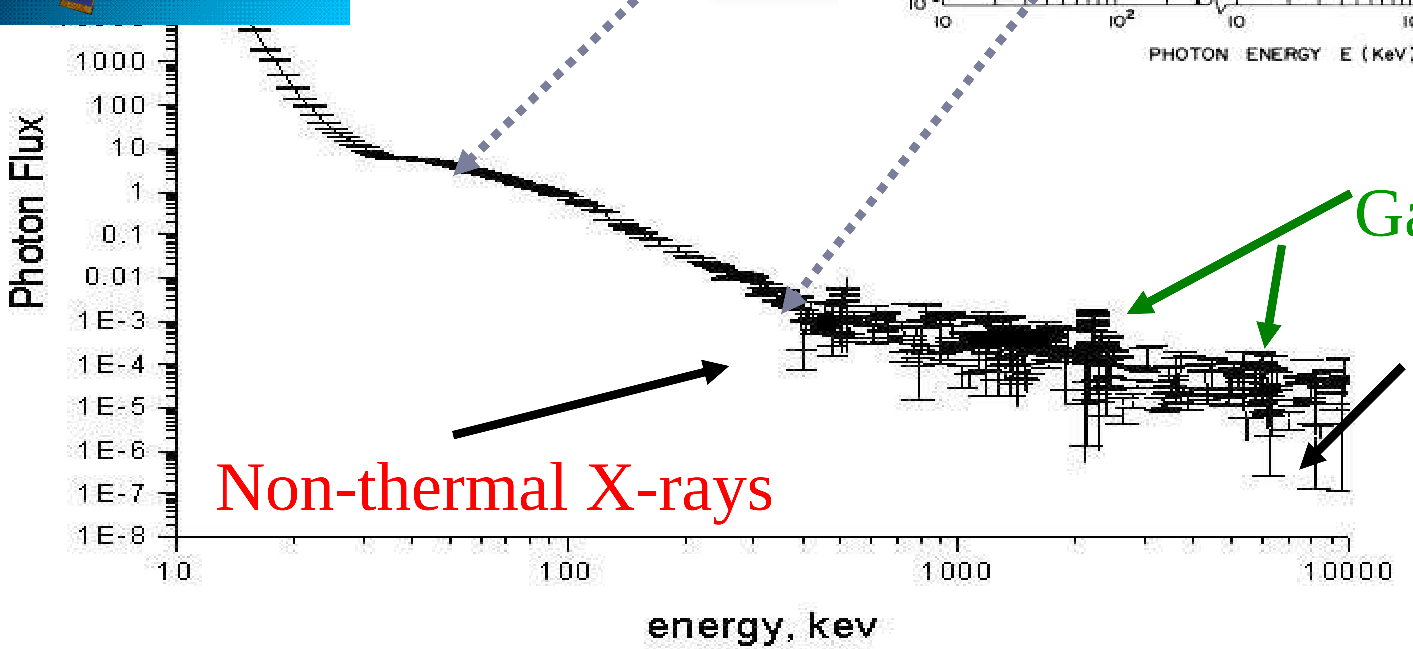
Angelos Vourlidas

Motivation – X-ray data

Pre-RHESSI X-ray measurements





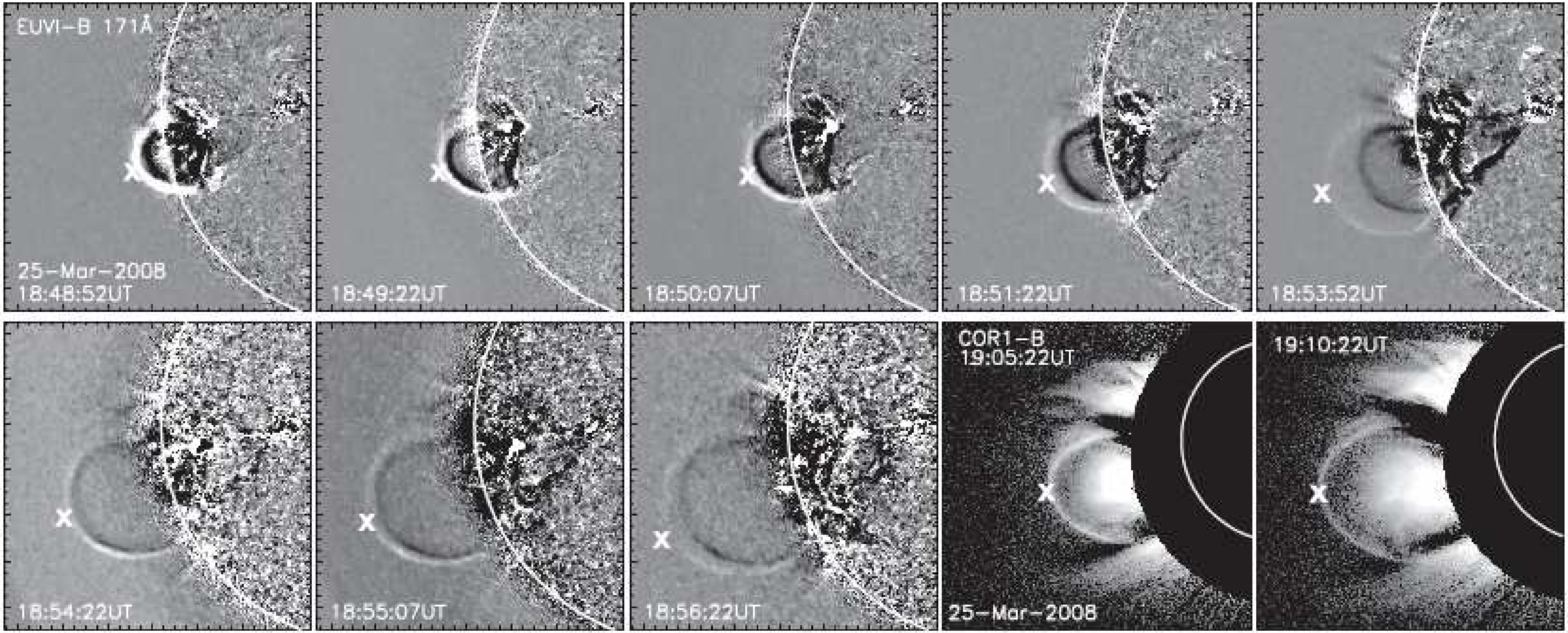


Ramaty High Energy Solar Spectroscopic Imager(RHESSI)

e.g. spectral index (*Kontar and MacKinnon, 2005*)

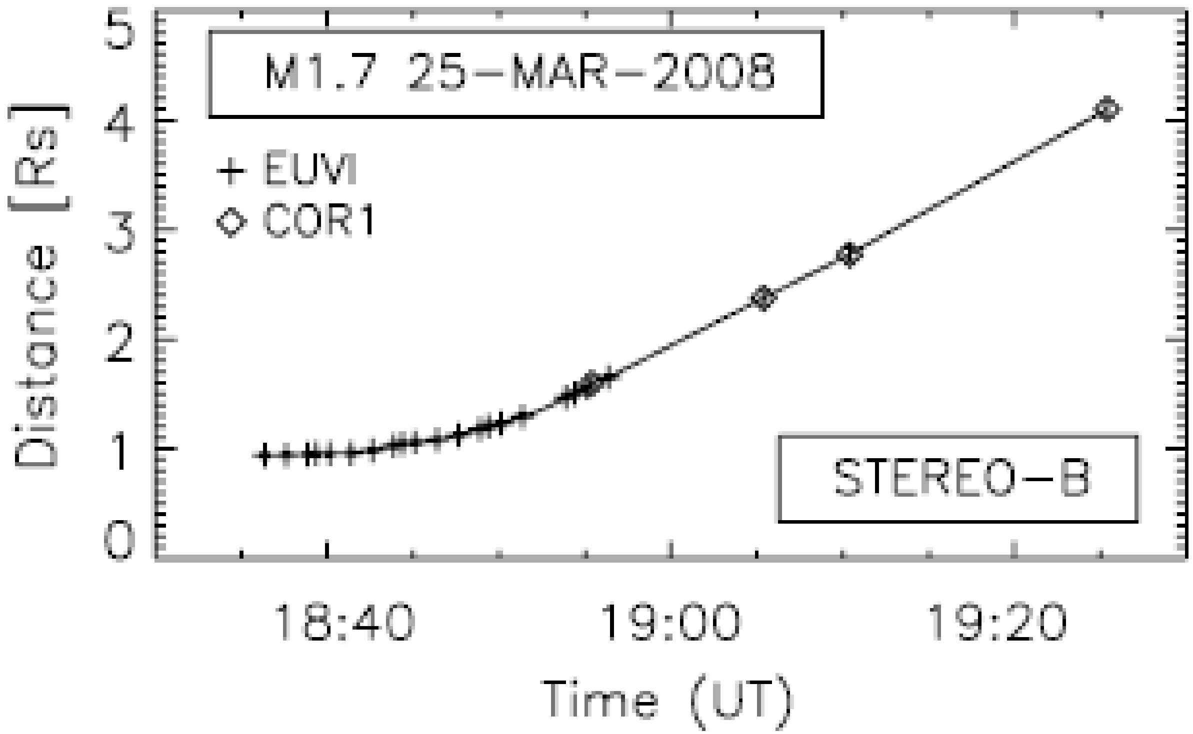
Height – time measurements

Let us consider CME propagation….



2008 March 25, M1.7 flare/CME event observed with *STEREO-B* from (Temmer et al, ApJ, 2010)

Height – time measurements



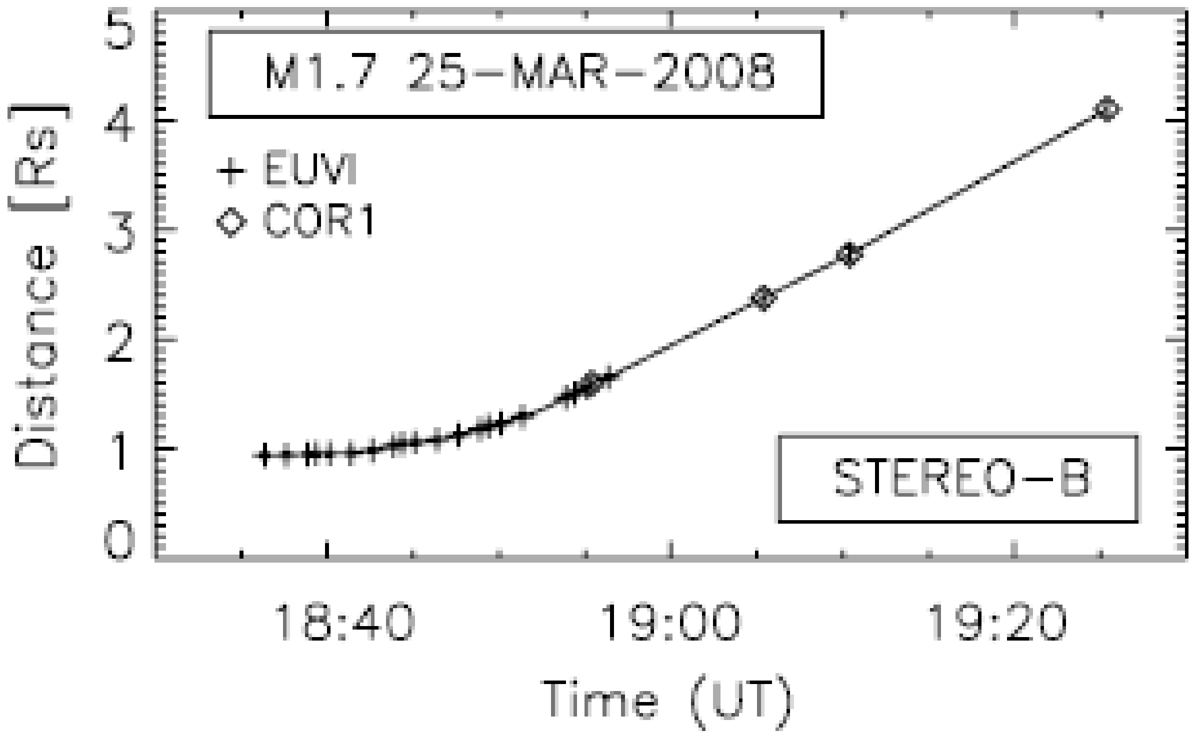
Distance vs time for 2008 March 25, M1.7 flare/CME event observed with *STEREO-B* from (Temmer et al, ApJ, 2010)

Two approaches to find velocity and acceleration of CMEs:

1) Forward fitting: to find the parameters of the model as a best fit to the original data

2) Model independent (**no model assumed**) inference of velocity and acceleration

Height – time measurements



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Two approaches to find velocity and acceleration of CMEs:

1) ~~Forward fitting: to find the parameters of the model as a best fit to the original data~~

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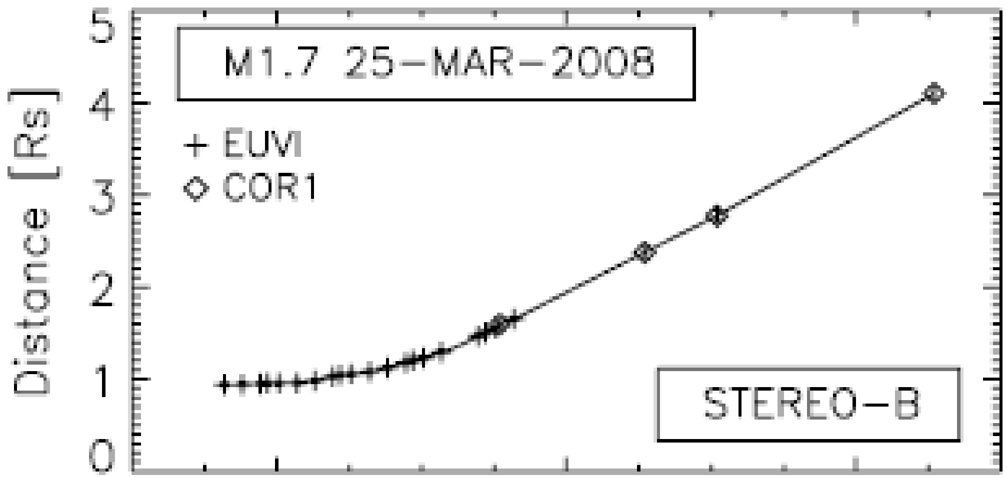
Height – time measurements

Let us assume that we have a analytical functions $h(t)$ over the finite time interval (t_0, t_N), while we are given a dataset of height measurements:



$h_i,\ i = 1\ldots N$

for a number of times

$t_i,\ i = 1\ldots N.$

The dataset has a finite uncertainty of the measurements δh, so that

$$h_i - h(t_i) < \delta h$$

Now our problem is to find the best smooth representations of

$$v(t)=\mathrm{d}h(t)/dt \quad (velocity)$$

$$a(t)=\mathrm{d}^2h(t)/dt^2 \quad (acceleration)$$

Derivative as an inverse problem

The problem of finding derivative can be written as the integral inversion problem (*Groetsch, C. W. 1984, Hanke, M., & Scherzer, O. 2001*)

Indeed, the height at a given time is given by an integral

$$h_i = h_0 + \int_{t_0}^{t_i} v(t')dt'$$

we can re-write this equation in the matrix form

$$\mathbf{h} - h_0 = \mathbf{S}\mathbf{v}$$

where **S** is the matrix representing our integral

h is the data-vector given $[h_1,\ldots,h_N]$,

v is the velocity-vector to be found $[v_1,\ldots,v_M]$,

Regularized solution of inverse problem

In other words we are looking for a solution of the minimization problem

$$||\mathbf{h} - h_0 - \mathbf{S}\mathbf{v}||^2 = min \qquad [1]$$

where $||\cdot||^2$ is a norm defined as $||\mathbf{h}||^2 \equiv \mathbf{h}^T\mathbf{h}$.

This problem [1] does not have a unique solution and additional constraints are needed (e.g. Berterro et al, 1985).

$\cancel{\mathbf{v}=\mathbf{S}^{-1}} \; (\mathbf{h}-h_0)$

Using Tikhonov **regularization** technique (*Tikhonov, 1963*), our problem becomes:

$$||\mathbf{h} - h_0 - \mathbf{S}\mathbf{v}||^2 + \lambda||\mathbf{L}\mathbf{v}||^2 = \mathbf{min} \qquad [2]$$

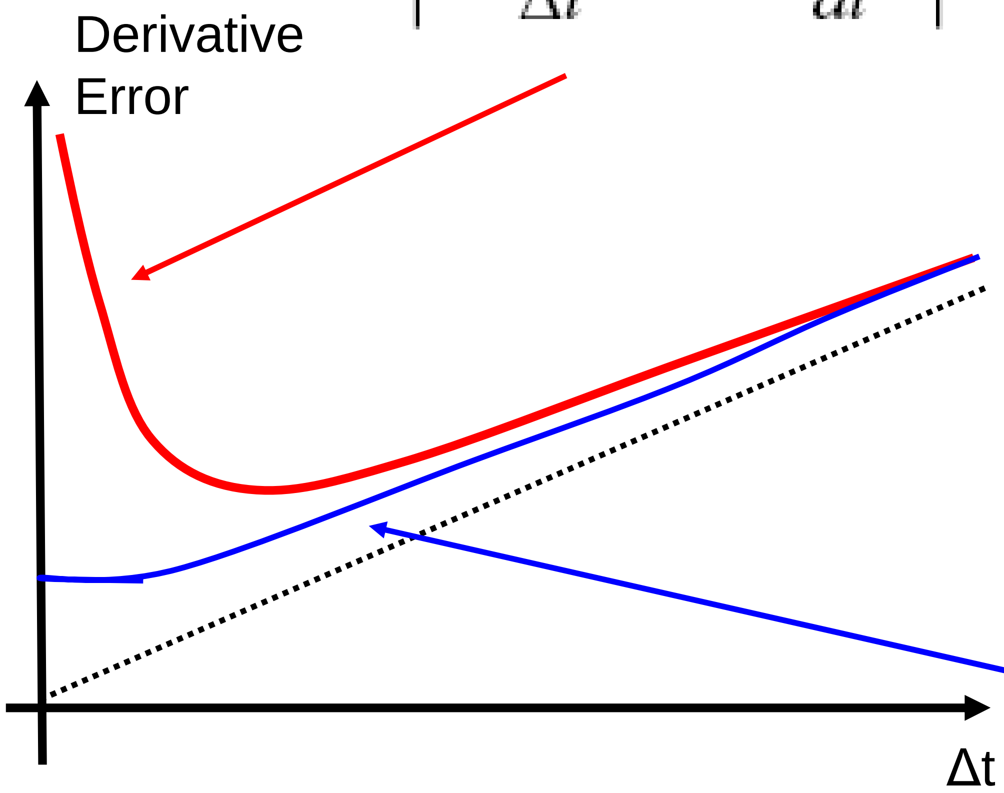
where **L** is the matrix representation of constraint operator, and λ is a regularization constant.

Importantly, the problem [2] is well-behaved and has a unique solution.

What is constraint matrix L?

The derivative error calculated from noisy data set:

$$\left|\frac{h_{i+1}-h_i}{\Delta t}-\frac{dh(t_i)}{dt}\right| \leq O\left(\Delta t+\frac{\delta h}{\Delta t}\right)$$



We will look for a function h(t) close to a given data set so that

$$\|\mathbf{h}-h_0-\mathbf{S}\mathbf{v}\|^2=\|\delta h\|^2$$

While the second derivative of h(t) or first derivative of v(t) has a minimum, the derivative error looks much better:

$$\left|\frac{h_{i+1}-h_i}{\Delta t}-\frac{dh(t_i)}{dt}\right| \leq O\left(\Delta t+\delta h\right)$$

Therefore, following *Hanke and Scherzer* (2001) we can choose $\mathbf{L}=\mathbf{D}_1$

The regularized solution

Hence we can write an explicit solution of minimization problem, which minimizes the amplification of the errors in the resulting estimate for the derivative, i.e. velocity:

$$\mathbf{v}_\lambda = \mathbf{R}(\mathbf{h} - h_0), \quad \text{where} \quad \mathbf{R} = (\mathbf{S}^T\mathbf{S} + \lambda \mathbf{D_1}^T\mathbf{D_1})^{-1}\mathbf{S}^T$$

The only unknown parameter is λ, which can be determined requiring the finite difference between our solution and the original dataset

$$||\mathbf{h} - h_0 - \mathbf{S}\mathbf{v}_\lambda||^2 = \alpha||\delta\mathbf{h}||^2$$

Parameter α tells us about the errors (should be around 1 in case of Gaussian errors)

The horizontal and vertical errors

Let assume that we know the true solution of our linear inverse problem v_{true}, then we can write

$$\mathbf{h} - h_0 = \mathbf{S}\ \mathbf{v}_{true} + \delta\mathbf{h}$$

The regularized solution of our inverse problem is

$$\mathbf{v} = \mathbf{R}\ (\mathbf{h} - h_0)$$

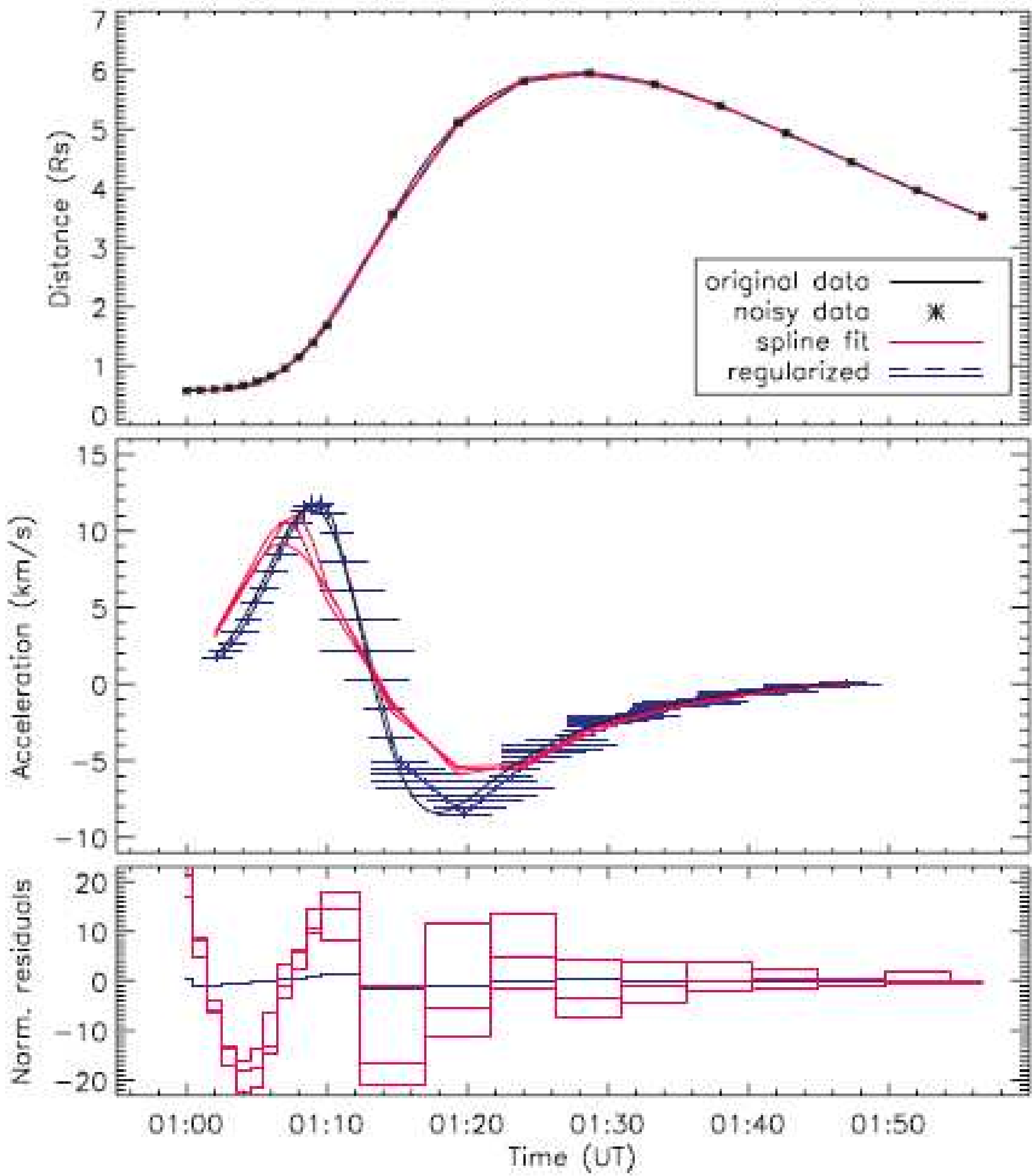
The difference between the true solution and our solution can be written as

$$\delta\mathbf{v} = \mathbf{v} - \mathbf{v}_{true} = (\mathbf{RS} - 1)\mathbf{v}_{true} + \mathbf{R}\delta\mathbf{h}$$

Resolution (horizontal error)

Data noise propagation error (vertical error)

Simulated data: example I

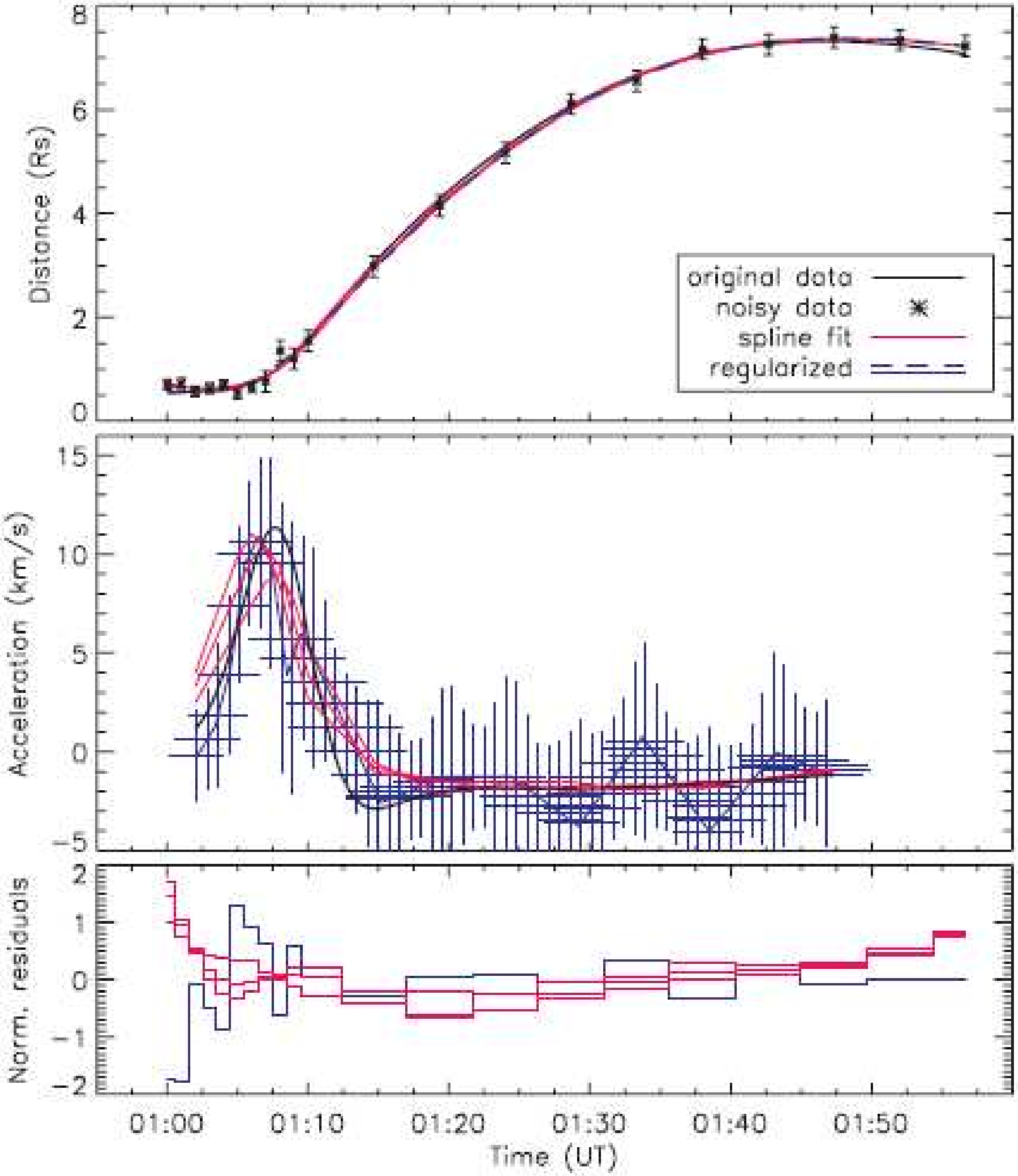


Simulated data (no noise added but discrete data set)

Acceleration profile (error due to discrete data set is evident)

Normalised residuals

Simulated data: example II

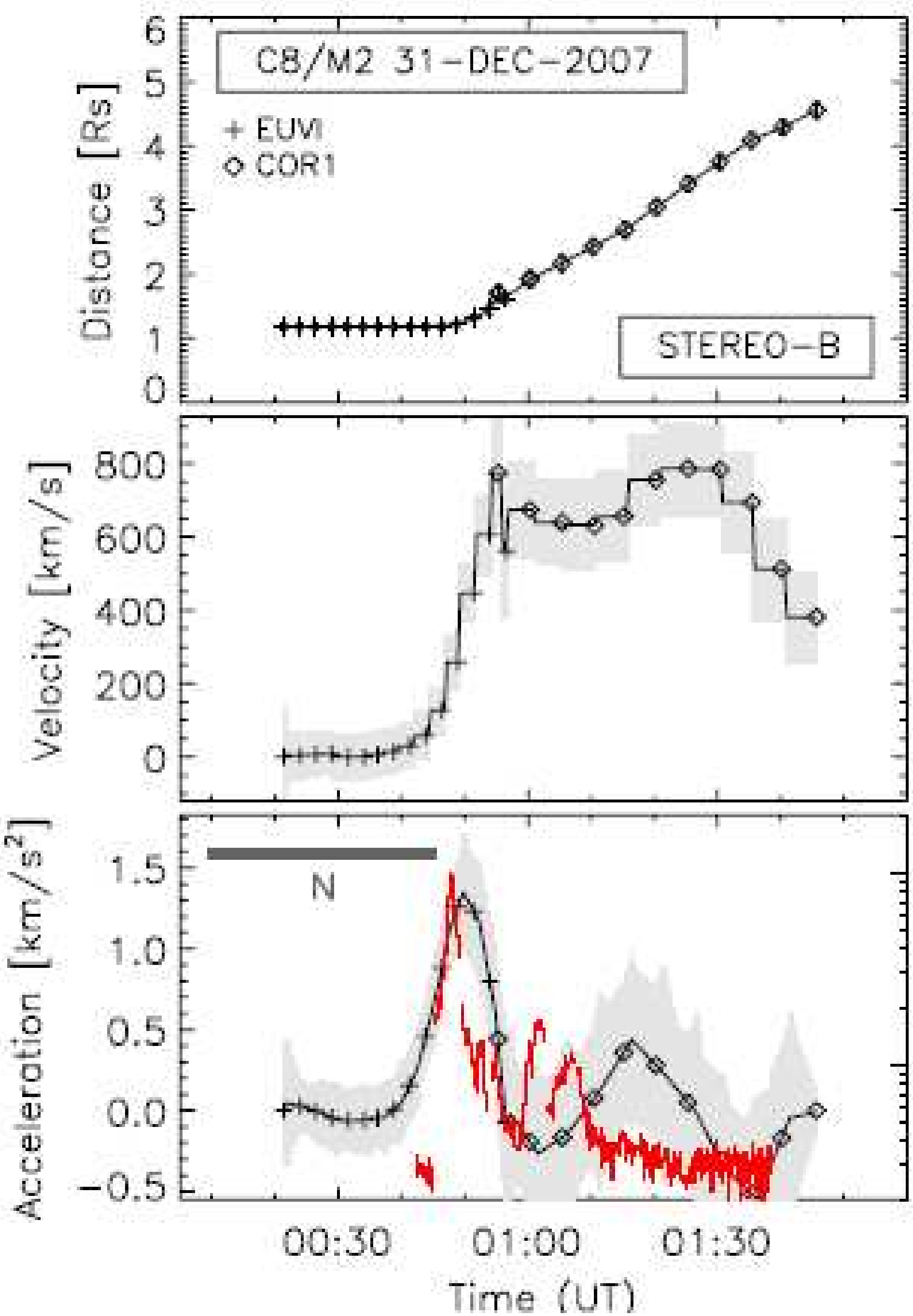


Simulated data (realistic noise added)

Acceleration profile

Normalised residuals

31-Dec-2007 flare/CME



Height-time data (Temmer et al, ApJ, 2010)

Velocity profile

Acceleration profile (Note change in acceleration profile)

Conclusions

Regularized inversion gives us **model-independent** (**without assumptions on functional shape**) **velocity and acceleration as a function of time**.

=>provides us with horizontal and vertical error bars and hence gives us confidence range for **velocity** and **acceleration**.

⇒Regularized derivative is IDL based package and easy to use

⇒Can be applied not only to CME data but to EIT waves etc